MILITARY PLANNING GAME

ROY E. HOPGOOD

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Submitted in partial fulfillment of
the requirements for the degree of

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ABSTRACT

The complex interrelationships among offensive, defensive, strategic, tactical, etc. weapon systems and the high cost of research and development, initial procurement, maintenance and operation of today's and the future's weapon complexes have forced decisionmakers to utilize the system analysis or cost-effectiveness approach. One educational device that is used to illustrate the principles of and the need for "Cost-Effectiveness" and "System Analysis" is a simulated military planning game. This paper developes several game form analytical models that can be solved by linear programming techniques. The objective is to use an analytical tool to illustrate and emphasize the impact and significance of the principles of systems analysis on military decisionmaking.

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CHAPTER I

INTRODUCTION

In the past, the choice among weapons was a somewhat isolated procurement decision. However, today the choices and the problems of military planning have become highly interconnected. This is a direct result of the rapid changes in technology and greater system complexity. Total cost and related time stream, lead-time and uncertainty factors in the R & D, procurement and operational phases add their impact. A weapon's dependence upon other weapons and supporting systems within the same service and in other services must be carefully considered by a decisionmaker. Thus systems analysis, cost-effectiveness and computational techniques are of necessity married to each other and to the military force-structure planning of the future.

As an instructional device, it is perhaps natural that the game form which requires its participants to base their decisions on overall dynamics of cost-effectiveness comparisons of competing weapon systems and uncertain actions of the enemy should be used to reflect the true spirit of systems analysis. A game of this type is intended to provide a simulated environment and a conceptual frame work within which to generate principles and concepts. Its prime purpose is pre-analytical. It may help to stimulate research, systematize issues, formulate problems more clearly, provide

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an initiative, preliminary or gross evaluation of alternative strategies or instrumentalities; but it is not usually expected to yield final answers.

In light of the foregoing, TEMPO (Economic Analysis Section) in 1961 developed a Military Planning Game to simulate the military planning and decision process at the highest level in the organization hierarchy. Its authors constructed the game so as to permit insights to be derived from the conditions of play. The follow principles are intended to be found in the course of play or in the post-game critique:

1. Current decisions must take future years into account. (Research and development actions imply procurement and operating expenses in later years.)

2. Systems planning can only take place in the contex of total force-structure planning.

3. The relevant length of the planning horizon is the indefinite future.

4. Estimates of effectiveness, costs, and timing are subject to considerable uncertainty - greater as the time increases between estimates and the actual period.

5. Obsolescence - a continuously critical consideration - is an increasing function of time.

6. Hedging policies involving a balance of forces may be quite desirable in view of uncertainties regarding future capabilities and intentions.

7. Because of budget constraints, technological progress and enemy-actions decision making becomes a delicate art which involves such contrasting considerations as the following:

    a. Older programs may never realize their expected

potential if newer and more promising ones are continually brought in to replace them.

b. Expenditures on marginal or submarginal programs may be worthwhile if they force the enemy to expend inefficiently a high enough proportion of resources to counter them.

c. Too thin a program spread may prove inefficient, and prevent fullest utilization of operational systems.

d. Certain generations of systems may have to be skipped.

e. Unless a substantial dominance in resources or planning is present, it is likely to be impossible to maintain a continuously leading position.

f. Delays in program implementation can lead to reduction of eventual utility.

8. Enemy actions should be systematically considered and related to the confidence in one's information on his actions.

9. The over-all problem of long-range military planning is too complex to attack without the aid of models and sophisticated research tools.(1)

This paper makes use of the TEMPO game frame-work to introduce a Linear Programming Technique that can be used to derive "best" feasible solutions under varying assumptions and thus serves to further demonstrate the effects of the principles listed above.

It should be noted, at this point, that the TEMPO game is a significant departure from reality in that each team starts with an identical force structure and has the opportunity to develope the same weapons throughout the play of the game. Therefore the Linear Programming Technique and a particular "best" feasible

solution have value only as a tool to illustrate the dynamic quantitative aspects of "systems analysis" and what happens when one tries to get the most for his money within a set frame of constraints. It is quite important to keep in mind that in the world of reality subjective reasoning must be applied to account for those areas of a problem which can not be quantified or where uncertainty dominates the assumptions made, in order to arrive at " a solution ". The analytical tools are intended to assist a decisionmaker by bounding those parts of the problem that can be so treated and thus narrowing the range within which " a feasible solution " exists. It is hoped that from the set of feasible solutions the " best " one can be picked.

CHAPTER II

GAME DESCRIPTION

TEMPO's Military Planning Game is played by two teams, which commence play with identical pre-specified force structures and defense budgets and each team has a choice of the same set of strategies. Thus the game is of the zero-sum, symmetric type.

The force structure is composed of two types of offensive systems identified here as 1 and 2 and two types of defensive systems identified here as 3 and 4. Defensive system 3 is only effective against offensive system 1 and like wise 4 is only effective against 2.

Each of the afore mentioned systems is composed of various weapons. Each weapon is characterized by :

a. having a specific number in inventory (operated or procured the previous year)

b. operating cost (annual)

c. procurement cost/time phase and limit (one time cost with a limit number of units that can be procured in one period to be operated in the following period)

d. R & D cost and time phase (annual R & D cost, time depending upon the number of periods over which the R & D is spread. Procurement of the weapon can not commence until the last period of R & D)

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e. utility value. (military worth)

The payoff is determined by the net effective offensive utility differences of the two teams.

A team's effective offensive utility value is found by taking the sum of the amounts by which its two offensive system utility values exceed the utility values of the oppositing team's defensive systems. (Note: Over defending has zero effect on payoff.) The game is played with a known probability of war. If war occures in one period, the budget of the team with least effective offensive utility value is docked in the subsequent period by the amount of the payoff. Total game payoff is summed at the end of the completed game.

At the beginning of each time period (year), each team makes a move (selects a strategy) constrained by a fixed current budget (only the trend of the future budget is known), the previous periods inventory and procurements, procurement limitations and new weapon availability via R & D. Each team draws upon the following quantitative or quasi-quantitative factors to compose their particular force mix.

a. They consider the relationships between R & D, procurement and operating cost, utility value and the system in which the weapons are designed to operate. Exact relationships are known at the time of the last R & D period and fixed therafter, until this point a degree of uncertainty exists.

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b. Intelligence procured in prior periods as to the force structure of the opponent.

c. Probability of war, which is known.

The teams submit their force mixes, a die is cast, if war is found to occur during that period the appropriate team's subsequent year budget is reduced by the amount of the payoff, if not each team is given the same fixed next period budget, intelligence information (if procured), weapon factor changes (for those in early R & D phases) and possiblly new weapons maybe introduced. The game continues thus until a predetermined (but unknown to the players) number of time periods have been played. Then, the post-game critique enters to bring out the principles of "system Analysis" that have been illustrated in the particular play of the game.

CHAPTER III

LINEAR PROGRAMMING TECHNIQUES

The TEMPO game solution, in general, is one where limited assets vary with time and the manner in which previously held assets were expended and where each team desires to maximize a total effect at the end of a specified or assumed time. It is a problem in resource allocation over time with all its aspects of cost-effectiveness. An attempt to discuss such problems in general would be presumptuous here; but the problem of resource allocation in the TEMPO game will be treated here making use of variations in linear programming techniques, in model form, to present some methodology and, it is hoped, some concepts which will be useful in dealing with or illustrating cost as related to measures of effectiveness. The game is of the zero-sum, symmetric type, in that each team starts with identical force structures and has available a choice of the exact same strategies to play under the same payoff rules. The payoff matrix is symmetric with zeroes down the diagonal. A "best" solution for one is also "best" for the other, so from here on we will treat the problem from one team's point of view, realizing that it also applies to the other team equally well.

1. Force allocation model -- 1

Given a weapon systems structure, including weapon types,

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costs, time phasing, and expected budget levels, the Force-allocation Model will determine the force mix over time which will give the maximum effectiveness for the fixed expected period budgets. Given actual or assumed weapon characteristics, a program determines the number of each weapon required in each time period to accomplish maximum cost-effectiveness, subject to the particular games constraints of dollars, procurement level, system mix, etc..

The procedure by which the preferred force mixes are calculated is based upon the fact that all the necessary characteristics of the problem can be expressed in terms of linear functions of the number of weapons in the inventory at any time.

The problem is to determine the force-mix which will maximize the objective function V_T - Total Utility Value which equals the sum of each operating weapon times its utility value summed over the entire period of play

$$V_T = \sum_{k=1}^{K} \left[\sum_{j=1}^{J} W_{1jk} \cdot U_{1jk} + \text{-----} + \sum_{j=1}^{J} W_{Ijk} \cdot U_{Ijk} \right] \tag{1}$$

Subject to the following constraints :

$$\sum_{i=1}^{I} W_{ijk} \cdot C_{oi} + P_{ijk} \cdot C_{pi} + R_{ik} \leq B_k \quad \text{for} \quad \begin{array}{l} j = 1,2,\text{---}J \\ k = 1,2,\text{---}K \end{array} \tag{2}$$

$$W_{ijk} \leq INV_i \qquad \text{for } k = 1 \text{ and } i = 1,2, ---I \quad (3)$$
$$j = 1,2, ---J$$

$$W_{ijk} \leq \left[W_{ijk(k-1)} + P_{ij(k-1)} - \overset{*}{P}_{ij(k-1)} \right] \quad \text{for } i = 1,2, ---I \quad (4)$$
$$j = 1,2, ---J$$
$$k = 23 \ ----K$$
$$i \text{ of } \overset{*}{P} \neq i \text{ of } W+P$$

$$P_{ijk} - \sum_{k=1}^{K} R_{ik} + \sum_{k=1}^{K} CR_{ik} \leq L_{ik} \qquad \text{for } i = 1,2, ---I \quad (5)$$
$$j = 1,2, ---J$$
$$k = 1,2, ---K$$

$$R_{ik} \leq CR_{ik} \qquad \text{for } i = 1,2, ---I \quad (6)$$
$$k = 1,2, ---K$$

$$\overset{*}{P}_{ijk} \leq W_{ijk} \qquad \text{for } i = 1,2, ---I \quad (7)$$
$$j = 1,2, ---J$$
$$k = 1,2, ---K$$
$$i \text{ of } \overset{*}{P} \neq i \text{ of } W$$

$$\sum_{i=1}^{I} W_{i3k} \cdot U_{i3k} \leq \sum_{i=1}^{I} W_{i1k} \cdot U_{i1k} \qquad \text{for } k = 1,2, ---K \quad (8)$$

$$\sum_{i=1}^{I} W_{i4k} \cdot U_{i4k} \leq \sum_{i=1}^{I} W_{i2k} \cdot U_{i2k} \qquad \text{for } k = 1,2, ---K \quad (9)$$

$$W_{ijk} \geq 0 \qquad (10)$$

$$P_{ijk} \geq 0 \qquad (11)$$

$$R_{ik} \geq 0 \qquad (12)$$

where

$i = 1,2,\ldots\ldots I \equiv$ the number of different weapon available.

$j = 1,2,\ldots\ldots J \equiv$ the number of different weapon systems.

$k = 1,2,\ldots\ldots K \equiv$ the number of time periods.

$W_{ijk} \equiv$ the number of the i^{th} type weapon allocated to the j^{th} system in the k^{th} period.

$U_{ijk} \equiv$ the utility value of the i^{th} weapon, in the j^{th} system in the k^{th} period.

$P_{ijk} \equiv$ the number of the i^{th} type weapon purchased in the k^{th} period for use in the j^{th} system.

$INV_i \equiv$ the number of the i^{th} type weapon in inventory at the beginning of a period of play.

$L_{ik} \equiv$ Purchase limit on the i^{th} type weapon in the k^{th} period.

$C_{oi} \equiv$ Unit operating cost of the i^{th} type weapon.

$C_{pi} \equiv$ Unit purchase cost of the i^{th} type weapon.

$CR_{ik} \equiv$ R & D cost of the i^{th} type weapon in the k^{th} period.

$B_k \equiv$ Actual or estimated budget for the k^{th} period.

$R_{ik} \equiv$ the amount of R & D purchased in k^{th} period for weapon i.

$P^*_{ijk} \equiv$ the asteriskdenotes that this weapon is a modification of another existing weapon.

The objective function equation (1) is in a more general form than called for by the game. In TEMPO's game the U_i's are constants, here the utility value of a weapon is a function of the system in which it is used and time. Note also that the game uses a particular

weapon in only one system.

The set of equations given by equation (2) specifies that in each time period total weapon R & D, procurement and operating costs can not exceed the budget for that period. Note that the model does not allow variation in the operating or procurement cost with time, the use of learning curves on cost, or the introduction of costs associated with disposal of weapons from inventory.

The set of inequalities given by equation (3) specifies the initial inventory for each weapon. Note that this set requires only that the initial inventory have an upper bound, thus allowing weapons to be removed from inventory if desirable.

Equation (4) is used to keep track of all non-purchasable weapons. With the variable definition used, these relations prevent the use of weapons in any future time period if they were not used or procured in the previous period. The equation also keeps track of those weapons which are modified into another weapon $\overset{*}{P}$.

Equation (5) defines the purchase constraints on each weapon as a function of time. It also is used to insure that the required periodic increments of R & D have been purchased. The set is more general than called for by the game where the purchase limit is a constant. However, in this way, the set can be used to specify the nonavailability of particular weapons, but a better way is not to define P for the weapon if no purchases are to be allowed.

Equation (6) specifies that the amount of R & D that can be expended on a weapon in a particular period is predetermined.

Equation (7) specifies that the number of weapons that can be modified is also limited by the number of operating weapons available to modify.

Equations (8) and (9) specify that in each time period defensive utility values of system 3 and 4 can not exceed the offensive utility values of systems 1 and 2. Note, while this eliminates the possibility of over defending for which there is no payoff, it might also mean reduced total utility if a particular defensive weapon has a very high cost-effectiveness relative to the weapons of the opposing offensive system.

Equation (10), (11) and (12) state that the variables R, W and P, R & D and the number of weapons operated, and procured, are always positive or equal to zero.

The model as defined above will give a team the "best" solution in the "greatest bang per buck" sense and is optimal only under the assumption that the probability of war is zero, and no intelligence exists about the opponent. However, the game is played with a known probability of war and a penalty payoff is imposed upon a losing team and intelligence can be procured. The liklihood that the model will give a team the maximum total utility decreases as the probability of war approaches unity and intelligence received indicates that the opponent

is not using " best " solutions. This serves to illustrate the necessity of considering not only long run economy of effort but also short run levels of effectiveness relative the opponents. Said in another way, total long run efficiency may have to be sacrificed in order to hedge against the probability of war.

The model can be used to test the sensitivity of a pay-off penalty being applied in a particular period simply by reducing the amount of the expected budget for that period. It also can be used to maximize the utility value at a particular period. In the process of doing this, it identifies the time-span critical R & D, investment and operating path.

2. Model modification -- 1

In order to bring in intelligence information and subjective reasoning equations (2), (8) and (9) can be replaced by the following equation:

$$\sum_{i=1}^{I}\left[W_{ijk}\cdot C_{oi} + P_{ijk}\cdot C_{pi} + R_{ijk}\cdot C_{Rik}\right] \leq B_{jk} \tag{13}$$

Where

$B_{jk} \equiv$ Estimated budget of the j^{th} system in time period k.

This equation will allow a team to constrain the amount of funds that can be expended on a particular system in each time period and thus apply selective subjective weighting into the model.

3. Model modification -- 2

The game and model as presently designed allow a particular weapon to operate in only one system with a constant utility value. Equation (3), (4) and (5) can be modified to:

$$\sum_{j=1}^{J} W_{ijk} \leq INV_i \qquad \text{for } k = 1 \text{ and } i = 1,2,\text{---}I \tag{3a}$$

$$\sum_{j=1}^{J} W_{ijk} \leq \sum_{j=1}^{J} \left[W_{ij}\,(k-1) + P_{ij}\,(k-1) + P^{*}_{ij}\,(k-1) \right] \tag{4a}$$

$$\text{for } i = 1,2,\text{---}I$$
$$k = 1,2,\text{---}K$$

$$\sum_{j=1}^{J} P_{ijk} - \sum_{k=1}^{K} R_{ik} + \sum_{k=1}^{K} C_{Rik} \leq L_{ik} \tag{5a}$$

$$\text{for } i = 1,2,\text{---}I$$
$$k = 1,2,\text{---}K$$

$$\sum_{j=1}^{J} P^{*}_{ijk} \leq W_{ijk} \qquad \text{for } i = 1,2,\text{---}I, \quad j = 1,2,\text{---}J \tag{7a}$$
$$k = 1,2,\text{---}K, \quad i \text{ of } P \neq i \text{ of } W$$

The above equations will further generalize the model to where any weapon can be used in any system and since U is already in generalized form, the game can be used to simulate the concept of tactics. This adds another dimension to the play of the game which is easy to handle here, however, it will considerably complicate manual play.

4. Model modification -- 3

To illustrate the effect of a forced "budget stream" in the force-structure in relation to level and total effectiveness over time,

equation (2) can be replaced by :

$$\sum_{i=1}^{I} \left(W_{ijk} \cdot C_{oi} + P_{ijk} \cdot C_{pi} + R_{ik} \right) \leq B_T \tag{14}$$

where

$B_T \equiv$ Total budget for K periods.

This equation relaxes the constraint of spending only a specified sum in a particular period, thus allowing dollars to flow into periods where an optimum payoff can be achieved. Note while total effective offensive utility will be greater than or at least equal to that obtainable under equation (2), in some periods of play (early ones) the effective utility maybe less. This serves to demonstrate that by accepting a relative low level of effectiveness in the early periods of weapons system development, a later stronger and cost wise more efficient structure can evolve. Again here the probability of war enters so one must, as discussed before, consider the level of effectiveness in each particular period.

5. Force allocation model -- 2

The TEMPO game could be modified so that the objective of the game was to see which team could maintain a fixed effectiveness level at minimum cost. For instructional purpose this would tend to emphasize the other way to approach a "system analysis" problem -- fix the effectiveness level that you want to obtain and then vary the other resources until a minimum cost mix is obtained.

The problem is to determine the force-mix which will minimize the objective function C_T -- Total cost for k periods which equals the total operating, procurement and R & D cost of each weapon employed during the period of play.

$$C_T = \sum_{k=1}^{K} \left[\sum_{j=1}^{J} W_{1jk} \cdot C_{o1} + --- + \sum_{j=1}^{J} W_{Ijk} C_{oI} + \right. \tag{15}$$

$$\left. \sum_{j=1}^{J} P_{1jk} \cdot C_{p1} + --- + \sum_{j=1}^{J} P_{Ijk} \cdot C_{pI} + \sum_{i=1}^{I} R_{ik} \right]$$

Subject to the following constraints :

$$\sum_{i=1}^{I} W_{ijk} = V_{jk} \qquad \text{for } i = 1,2, --- J \qquad k = 1,2, --- K \tag{16}$$

and also prior constraints (3), (4), (5), (6), (7), (10), (11) and (12).

where

$V_{jk} \equiv$ The number of the j^{th} type weapons required to equal a specified utility value - V_{jk} for the j^{th} system in period k.

The set of equations given by Equation (16) specifies that in each time period each system must have a utility value equal to some predetermined amount - V_{jk}. As to the other constraints, the aforementioned comments also apply here.

The model as defined above can be used to sence the effect of requiring a specified military posture. The cost in each period, total

cost and the structure of the cost-time-stream are the interesting relations to derive, compare and discuss in the light of particular or comparative subjective strategies.

CHAPTER IV

GAME EXAMPLE

This chapter will display various solutions to a cut down version of the Tempo game. The force structure which follows is composed of eight weapons types, four weapon systems and is played over a ten year period.

1. Weapon characteristics

Weapon Name	I	J	U	Co	Cp	L
Flood	1	1	20	30	50	25
LeMay	2	1	500	150	250	15
Decker	3	2	40	50	70	25
Schriever	4	2	150	70	120	15
Vinson	5	3	50	60	100	25
Super-Vinson*	6	3	80	80	50	25
Mahon	7	4	15	20	40	25
Lemnitzer	8	4	110	60	90	25

* Modifies existing Vinson.

where

$I \equiv$ Weapon Number

$J \equiv$ Weapon System

$U \equiv$ Utility Value

$C_o \equiv$ Yearly Operating Cost

$C_p \equiv$ Procurement Cost

$L \equiv$ Yearly Procurement Limit

2. Initial inventory

Weapon Name	Initial Inventory
Flood	70
Decker	40
Vinson	20
Mahon	40

3. Budget

Year	Budget	Year	Budget
1	6,300	6	9,000
2	6,700	7	8,500
3	7,000	8	8,100
4	7,600	9	7,900
5	8,200	10	7,500
		Total	76,800

4. R & D requirements

Weapon Name	C_{R1}	C_{R2}	C_{R3}	C_{R4}	C_{R5}	C_{R6}
LeMay	400	900	700			
Schriever			300	500	300	200
Super-Vinson	200					
Lemnitzer	400	400				

where

$C_{R1} \ldots C_{R6} \equiv$ R & D cost in 1st...6th period of play

5. Force allocation model-1, (refer p.8) Computed detail results.

Control Data's CDm2/Linear Programming System was used to compute the following results. The system uses the Revised Simplex method in which the inverse is in product form. It can handle problems up to 200 row constraints and 599 column variables. The game example presented here has 142 column variables with 187 row constraints. The computer running time was approximately 3 minutes.

Table I, force-mix gives an exact brake down as to which weapons are operated - W, procured - P and R & D expenditure - R during each period of play. Note that this particular solution is in no way "intuitively obvious", but its trend is. The less cost-effective weapons are gradually replaced with the more cost-effective ones. In the case of weapon 8 it even pays to procure 3.8 units in period 7 to operate only during periods 8 and 9. This result can be traced to the decreasing budget trend that commenced in the 7th period.

The utility values for each system, by year, their totals and net offensive (payoff) are set down in Table - II. Note that the weapons available to defensive system 3 are not cost-effective after the second period of play. Total yearly utility decreases in the second year, stays low in the third year and then rises to a maximum in the last year. This result can be traced to the heavy R & D investment in the early years. Procurements in the middle

years were taken care of by relatively higher budgets. In the later years lower relative budgets were able to maintain high utility value, due to the previously sunk R & D and procurement cost on the more efficient operating weapons.

TABLE - I

FORCE - MIX

Year	1			2			3			4			5		
Weapon \ Variable	W	P	R	W	P	R	W	P	R	W	P	R	W	P	R
1	50.8			21.3											
2			400			900		15	700	15	15		30	15	
3	40	1.8		41.8			41.8			41.8	3.4		45.2	1.5	
4									300			50			300
5	20			8.5											
6															
7	40			40											
8			400		15.2	400	15.2			15.2	1.2		16.4	.5	

Year	6			7			8			9			10		
Weapon \ Variable	W	P	R	W	P	R	W	P	R	W	P	R	W	P	R
1															
2	45	5.3		50.3			50.3			50.3	1.0		51.3		
3	46.7														
4		15	200	15	15		30	8		38			38		
5															
6															
7															
8	16.9	3.8		20.7	20.4		41.3			41.3			37.5		

TABLE - II

UTILITY VALUES

System / Year	1	2	3	4	Total	Net. Off
1	1160	1600	1000	600	4360	1160
2	425	1675	425	600	3125	1075
3	0	1675	0	1675	3350	0
4	1950	1675	0	1675	5300	1950
5	3900	1810	0	1810	7520	3900
6	5850	1870	0	1870	9590	5850
7	6520	2250	0	2250	11020	6520
8	6520	4500	0	4500	15520	6520
9	6520	5700	0	4525	16745	7695
10	6650	5700	0	4125	16745	8225
Total	39495	28455	1425	23630	93005	42895

6. Comparative solutions of various modifications to the Basic Model.

TABLE III

TOTAL UTILITY VALUES PER YEAR

Year	A	B	C	D	E	F	G
1	4360	4260	4260	1550	0	0	4135
2	3125	2560	2560	1550	0	0	3230
3	3350	2750	2750	1550	2750	2750	3575
4	5300	6530	6530	3500	7450	7450	5490
5	7520	9520	9520	5450	12150	12150	7035
6	9590	11850	12925	7400	15125	15800	7890
7	11020	14725	15620	13640	17375	18050	11100
8	15520	16900	16820	20310	19625	18050	13375
9	16745	17165	16470	21780	19625	18050	14355
10	16475	16315	15700	21780	14850	18050	14460
Total	93005	102570	103155	98510	108950	110350	84645

where

A $\equiv$ The Basic Model (refer p. 8)

B $\equiv$ The Basic Model with constraints (8) and (9) applied only in the 10th period (refer p. 10)

C $\equiv$ The Basic Model without constraints (8) and (9)

D $\equiv$ Model modification -- 3 (refer p. 15)

E $\equiv$ Model modification -- 3 with constraints (8) and (9) applied only in the 10th period

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F $\equiv$ Model modification -- 3 without constraints (8) and (9)

G $\equiv$ Model modification -- 1 where system 1 and 2 were each allocated 3/10 of each yearly budget and system 3 and 4 were allocated 2/10 (refer p. 14)

Since many relationships are involved, only a few explanatory remarks concerning the data in Table - III will be made here. First, comparing the Basic model - A with B (defensive system utility value can not exceed comparable offensive system in the last period of play) and C (no restrictions on defensive systems relative to offensive) the total utility increases, as would be expected, when going from A to B to C. The same results occure when going from D to E to F with the total cost model. Also one would expect G to have the lowest number of utility points since money is forced into a system whether it is efficient or not. Total utility values of D, E and F are greater than their respective counterparts A, B and C due to the fact that the latter plays have a yearly budget restriction and thus force spending on early weapons which yield less utility per dollar. However, one should not over look the fact that A, B and C give a greater degree of protection during the first years of play.

TABLE IV

NET OFFENSIVE UTILITY VALUES (PAYOFF)

Year	A	B	C	D	E	F	G
1	1160	1160	1160	10	0	0	550
2	1075	1560	1560	10	0	0	1560
3	0	0	0	10	0	0	270
4	1950	1950	1950	1960	1950	1950	0
5	3900	3900	3900	3910	3900	3900	1280
6	5850	5850	4625	5860	5850	5850	1920
7	6520	6800	5070	8020	5850	5850	2630
8	6520	6800	5070	9750	5850	5850	4325
9	7695	6800	5070	11220	5850	5850	5445
10	8225	6915	5070	11220	5850	5850	5960
Total	42895	42535	33475	51970	35100	35100	23940

In general the yearly trend of the net offensive utility values (Table IV) follows an early period decline reaching a minimum in the third year and then increasing to a maximum in the last year. This is due to the same factors mentioned in the discussion of Table II above. A result that is expected but not so obvious is that net offensive payoff decreases as you proceed from A to B to C and from D to E to F, while total utility (Table III) increases. This is due to the existence of a defensive weapon that is more cost-effective than any counterpart offensive weapon and the relaxing of the constraints applying to over-defending.

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The maximum utility value that can be obtained in the 10th period (total cost model, Model modification -- 3 (refer p. 15))

a. with constraints (8) and (9) <u>31,127</u>

b. with constraints (8) and (9) in 10th period <u>31,213</u>

c. without constraints (8) and (9) <u>31,357</u>

Force allocation model -- 2, (refer p. 16) effectiveness levels of 3,000 utility per year for systems 1 and 2 and 2,000 for system 3 and 4 were set. The minimum cost required to keep this effectiveness level over the ten year period of play was 102,416 dollars. The ten year utility in this case was 100,000 points. When effectiveness was not held constant, approximately 100,000 utility points were obtained from a budget of 76,800 dollars. This clearly shows the price that must be paid to maintain fixed effectiveness levels when total utility per dollar is increasing over time.

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